

Integrable Systems Seminar:

CH HIERARCHY

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Abstract:

This is the 2nd one of the two talks for the CH series. My presentation is based on my 2003 paper (see references below). I will show that the Camassa-Holm (CH) spectral problem yields two different integrable hierarchies of nonlinear evolution equations (NLEEs), one is of negative order CH hierarchy while the other one is of positive order CH hierarchy. The two CH hierarchies possess the zero curvature representations through solving a key matrix equation. We see that the well-known CH equation is included in the negative order CH hierarchy while the Dym type equation is included in the positive order CH hierarchy. Furthermore, under two constraint conditions between the potentials and the eigenfunctions, the CH spectral problem is cast in: 1. a new Neumann-like N -dimensional system when it is restricted into a symplectic submanifold of R^{2N} , which is proven to be integrable by using the Dirac-Poisson bracket and the r-matrix process; and 2. a new Bargmann-like N -dimensional system when it is considered in the whole R^{2N} , which is proven to be integrable by using the standard Poisson bracket and the r-matrix process. Associated with the integrability, I will show two 4×4 instead of $N \times N$ r-matrix structures. One is for the Neumann-like system (not the peaked CH system) related to the negative order CH hierarchy, while the other one is for the Bargmann-like system (not the peaked CH system, either) related to the positive order hierarchy. The whole CH hierarchy (an integro-differential hierarchy, both positive and negative order) is shown to have the parametric solutions which obey the corresponding constraint relation. In particular, the CH equation, constrained to a symplectic submanifold in R^{2N} , and the Dym type equation have the parametric solutions. Moreover, we see that the kind of parametric solution of the CH equation is not gauge equivalent to the peakons. Solving the parametric representation of the solution on the symplectic submanifold gives a class of a new algebro-geometric solution of the CH equation.

Finally, I will propose an open problem to the Dym type equation.

References:*/ Zhijun Qiao/* (2003): The Camassa-Holm hierarchy, N -dimensional integrable systems, and algebro-geometric solution on a symplectic submanifold, *Communications in Mathematical Physics* **239**, 309-341 (PDF file <http://www.math.panam.edu/%7Eeqiao/Qiao-2003-Aug-CMP.pdf>).